LA-2811 Addendum

LOS ALAMOS SCIENTIFIC LABORATORY

OF THE UNIVERSITY OF CALIFORNIA ○ LOS ALAMOS NEW MEXICO

OTHER DETAILS OF POST-FISSION BETA DECAY

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OF THE UNIVERSITY OF CALIFORNIA LOS ALAMOS NEW MEXICO

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OTHER DETAILS OF POST-FISSION BETA DECAY

by

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ABSTRACT

In this report are summarized various detailed numerical results obtained in calculations reported in LA-2811, but not specifically relevant to the purpose of that report. These include the time dependence of average beta disintegration energies, and the average number of beta decays for various nuclides, as well as plots of these quantities at certain specific times as a function of the single parameter $\bar{z}$, which characterizes the fissioning nuclide. It is hoped that the latter will be useful for extending the results to other nuclides not calculated explicitly.

ACKNOWLEDGMENTS

The author is grateful to his colleagues, Drs. H. Hoerlin and C. Longmire, for pointing out the potential utility of the present report to research in various areas.

I. INTRODUCTION

In a previous report[1] a model was constructed to simulate the post-fission beta decay process in terms of average properties of the fission fragment families. Various parameters in this model were chosen once and for all from independent data, and two parameters were chosen to fit data on the beta decay rate[2] of $U^{235}(n,f)$ and the post-beta-decay gamma emission rate[3] of the same nuclide. One parameter, $\bar{z}$, was left to characterize the fissioning nuclide in terms of the average displacement of its prompt fragments from the line of stability. A prescription was provided for determining this parameter for any nuclide at any excitation energy if only a knowledge of ν, the number of prompt neutrons emitted in fission, was available at some excitation energy for some isotope of the fissioning species in question. The model so prescribed, together with the prescribed estimation of $\bar{z}$, combine to provide at short times ($t \leqq 10$ seconds) an adequate theoretical description of all the available data[2,3,4] on the subject. One is therefore encouraged to utilize the model for extrapolative or interpolative estimation of situations not yet studied in the laboratory.

To facilitate such estimation, this report provides a fuller account of the numerical results than was considered relevant to the specific goals of the preceding report. The present summary should therefore be considered an addendum to LA-2811. In particular, the results presented here are subject to the various limitations and approximations emphasized in LA-2811, but not re-itemized here.

II. SUMMARY OF CONTENTS

Table I lists the nuclides calculated explicitly and the corresponding values of $\bar{z}$, together with the experimental situation these values were chosen to reproduce. Then follow graphical presentations of the calculated rate, $\bar{\lambda}$, of beta decay; of the average disintegration energy rate, $\dot{E}_d$; and of the average gamma-energy rate following beta decay for each of these nuclides. Subsequent figures provide plots of these rates against $\bar{z}$ at various specific times. These figures could be utilized to interpolate to different excitation energies and different fissioning species.

III. SPECIFICATION OF PARAMETERS

For the convenience of the reader, we describe again here the various parameters of the model, together with their numerical values and the manner and source from which each was obtained.

(1) The constant c_1 is the coefficient of the odd-A mass parabola. The value 1.61 mc^2 was chosen as an average over the region of A appropriate to fission fragments of the values given in reference 5.

(2) The half width, δ, of the assumed Gaussian distribution in charge of fission fragments of a given mass was taken equal to 1.0 from the work of reference 6.

(3) The even-odd mass difference, Δ, was taken as an average over the fission fragments from reference 5.

(4) The average gamma energy, E_γ^o, in excess of the minimum characteristic of the type of beta decay was chosen to be 1.03 MeV by fitting it and the parameter c_2 (below) to the measured beta-decay rates[2] and gamma-energy rates[3] for $U^{235}(n,f)$.

(5) The constant c_2 specifies the average beta-decay matrix element. It was chosen simultaneously with E_γ^o to fit the U^{235} data mentioned. The numerical value is 3.25×10^{-6}/sec and corresponds to a log ft of about 4.5 for the average beta decay.

(6) The parameter $\bar{z}$ characterizes both the fissioning nuclide and its excitation energy in the present model. It was calculated for U^{235} with $\nu = 2.50$ to have the value 3.54. For other nuclides and other excitation energies (with different ν values), $\bar{z}$ is estimated perturbatively as detailed in reference 1. A more detailed discussion of these parameters and the model they characterize can be found in reference 1.

IV. EXTENSIONS TO OTHER KINDS OF FISSION

Besides neutron-induced fission, the present results can be applied to other types of fission, such as A(n,nf) and A(γ,f), A(γ,xnf), by setting up a prescription for obtaining the appropriate $\bar{z}$. One basis for such a prescription is the observation[1] that $\bar{\nu}$ appears to be approximately the same for all members of an isotopic family _at the same neutron bombarding energy_. The underscoring is essential, since they provide an automatic recipe for including, at least approximately, odd-even binding effects. This prescription, together with the assumption that a post-fission neutron (which is estimated[1,7] to carry off 7 MeV of energy) is exactly equivalent to a pre-fission neutron, allow one to treat A(n,xnf) processes for all x as a single problem merely by adjusting ν (and thus $\bar{z}$) to the value appropriate for the incoming neutron involved. By looking up the relevant neutron binding energy and converting a photon absorption thereby to an equivalent neutron process, A(γ,xnf) processes can be similarly capsulized.[8]

The above procedures are somewhat crude. In particular the energy associated with a pre-fission neutron emission certainly depends on the nuclear excitation energy, if only weakly. Nor, indeed, can the simple assumption that each neutron costs exactly 7 MeV of energy be considered any more than a first approximation. Nonetheless, accumulated errors corresponding to an error of a full MeV in the equivalent excitation energy would shift $\bar{z}$ by only about 0.03. The corresponding change in the calculated quantities would typically be less than experimental errors in comparable measurements presently available.

V. DEFINITIONS OF CALCULATED QUANTITIES

We present here a precise specification of the calculated quantities presented in the following pages. For convenience we recall that a given beta decay is specified in reference 1 by two indices, τ and σ, which indicate its type, and a subscript, j, which denotes how far removed it is from the line of stability. The index τ assumes three values denoting whether it belongs to an odd mass chain ($\tau = 0$) or to an even mass chain with the nuclide nearest to $\bar{z}$ being even-even ($\tau = 1$) or odd-odd ($\tau = 2$). The corresponding weights are $g^{\tau} = \frac{1}{2}, \frac{1}{4}, \frac{1}{4}$, respectively. The index σ also assumes three values, corresponding to the three types of decay: $(o,e) \rightarrow (e,o)$, $\sigma = 0$; $(e,e) \rightarrow (o,o)$, $\sigma = 1$; and $(o,o) \rightarrow (e,e)$, $\sigma = 2$. Clearly these indices are partially redundant, since $\tau = 0$ necessarily implies $\sigma = 0$. The definition was chosen for convenience in the numerical computations, and one understands that $\sum_{\sigma\tau}$ is to be carried out with due cognizance of such limitations.

One then defines

$$\dot{E}_{\gamma}(t) = \sum_{j\tau\sigma} \lambda_j^{\tau\sigma} P_j^{\tau\sigma}(t)\, g^{\tau} E_{\gamma}^{\sigma} = \text{"EDOT(GAMMA)"} \qquad (1)$$

where $\lambda_j^{\tau\sigma}$ is the average beta-decay rate for decays of the type (τ,σ) in the j^{th} group from stability; $P_j^{\tau\sigma}$ is the calculated population of that type in that group at time t; g^{τ} is the weight function mentioned above; and E_{γ}^{σ} is the average gamma energy following decays of type σ.

This quantity is plotted in units of MeV per second per fission.

Next one defines

$$\bar{\lambda}(t) = \sum_{j\tau\sigma} \lambda_j^{\tau\sigma} P_j^{\tau\sigma}(t)\ g^{\tau} = \text{"LAMBDA"} \tag{2}$$

equal to the average number of the beta decays per second per fission. Finally, the rate of energy emission from beta decay is given by

$$\dot{E}_d(t) = \sum_{j\sigma\tau} \left[(w_j^{\sigma})^2 + 1\right]^{\frac{1}{2}} \lambda_j^{\tau\sigma} P_j^{\tau\sigma}(t)\ g^{\tau} = \text{"EDOT(D)"} \tag{3}$$

This is the total (including the rest mass of the electron) average rate of generation of energy by beta decay at time t. It is plotted in units mc^2 per fission per second to emphasize the fact that the electron rest mass is included.

Clearly the difference,

$$\dot{E}_d(t) - \bar{\lambda}(t) = \dot{E}_\beta(t) \tag{4}$$

is the rate of emission of energy associated with electrons and antineutrinos, excluding the rest mass, if the reader should require this quantity. Equation (4) differs from the definition of equation (11), reference 1, which is erroneous.

VI. DISCUSSION OF ILLUSTRATIONS

Figures 1 through 15 present the calculated time dependence of the three quantities, $\bar{\lambda}$, $\dot{E}_d$ and $\dot{E}_\gamma$, for each of the targets listed in Table I and studied in references 1 and 3.

Figures 16 through 18 present the same calculated results as a function of $\bar{z}$, with time as a parameter. These figures are designed to facilitate estimations for target nuclides and experimental circumstances other than those calculated explicitly. They are simply smooth-curve interpolations in $\bar{z}$ of the numerical results in figures 1 through 15.

TABLE I

Average Displacements, $\bar{z}$[*]

Target	E_n	ν	$\bar{z}$
U^{235}	G(1.47)	2.58	3.52
	2.00	2.80	3.47
U^{233}	G(1.47)	2.70[**]	3.05
U^{238}	G(1.47)	2.82	4.12
Th^{232}	1.60 MeV	2.08[***]	3.97
Pu^{239}	G(1.47)	3.06	3.29
	2.10	3.12	--

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[*] This table presents the values of $\bar{z}$ used in the calculations, together with ν-values used to obtain them from the U-235 value via Eq. (19) in reference 1. The second column indicates the neutron energy, or a G in the case of a measurement in the reactor Godiva[7] with the mean energy of the Godiva spectrum in parentheses.

[**] This value of ν is obtained by adding to the value measured in the reactor Topsy the difference between the Godiva and Topsy measurements for U-235.

[***] This value of ν is obtained by extrapolating via Eq. (19) in reference 1 from data[7] at 3.5 MeV to the indicated (fission threshold) neutron energy.

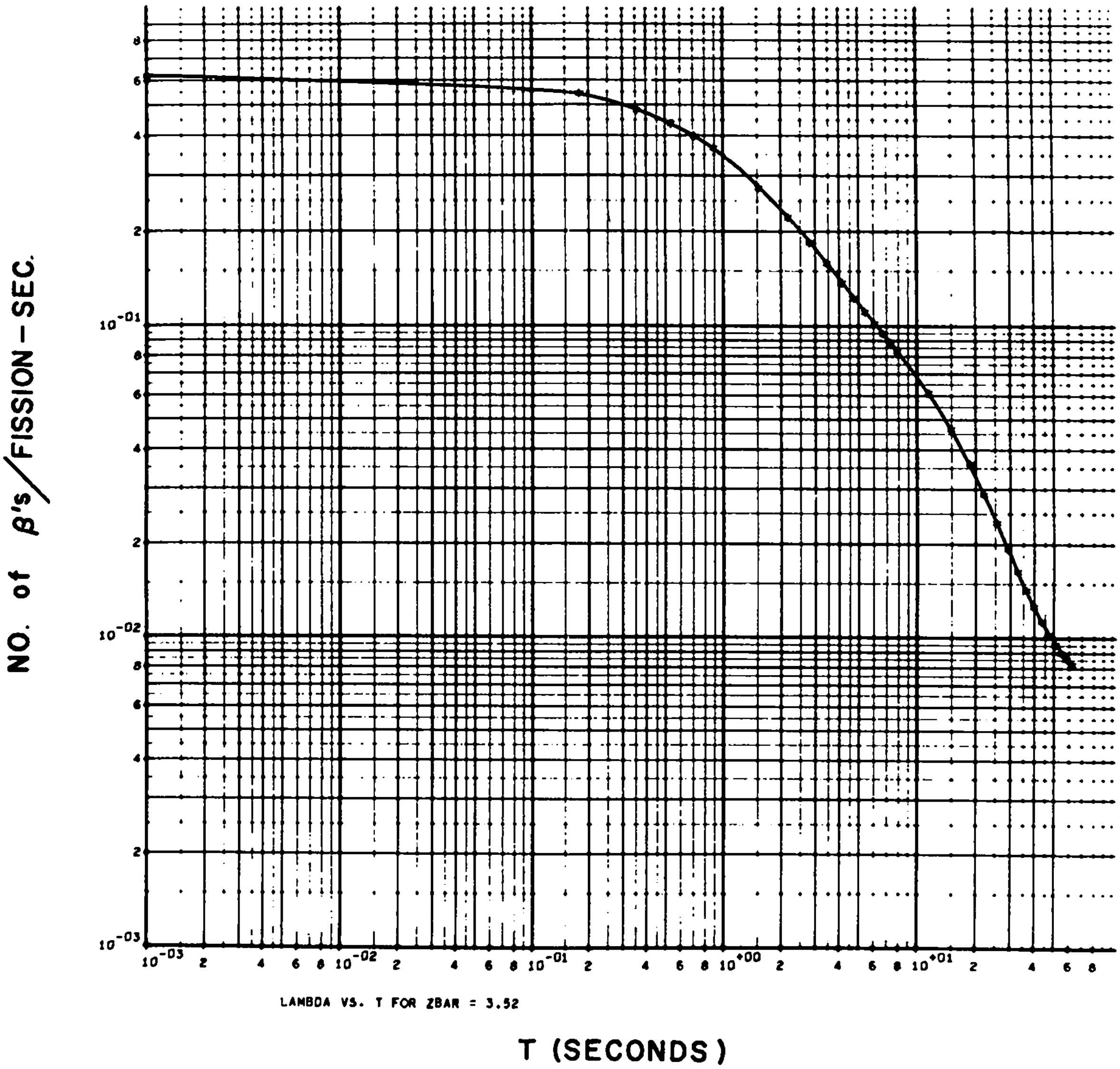
NO. of β's/FISSION-SEC.
10^-01
10^-02
10^-03
8
6
4
2
10^-03
10^-02
10^-01
10^+00
10^+01
LAMBDA VS. T FOR ZBAR = 3.52
T (SECONDS)

Fig. 1. U^{235} + n.

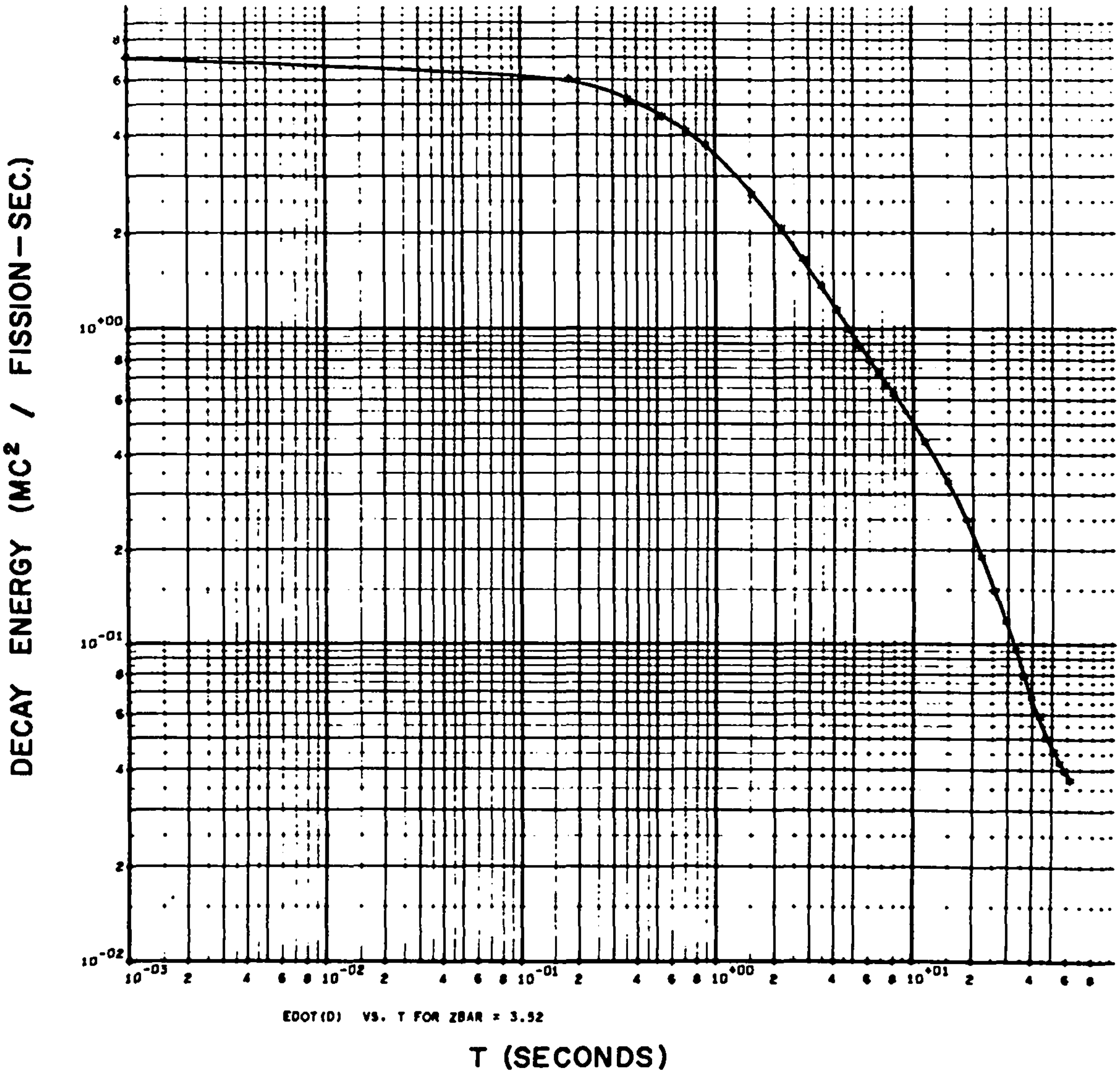
DECAY ENERGY (MC² / FISSION-SEC.)
10⁺⁰⁰
10⁻⁰¹
10⁻⁰²
2
4
6
8
10⁻⁰³
10⁻⁰²
10⁻⁰¹
10⁺⁰⁰
10⁺⁰¹
EDOT(D) VS. T FOR ZBAR = 3.52
T (SECONDS)

Fig. 2. U^{235} + n.

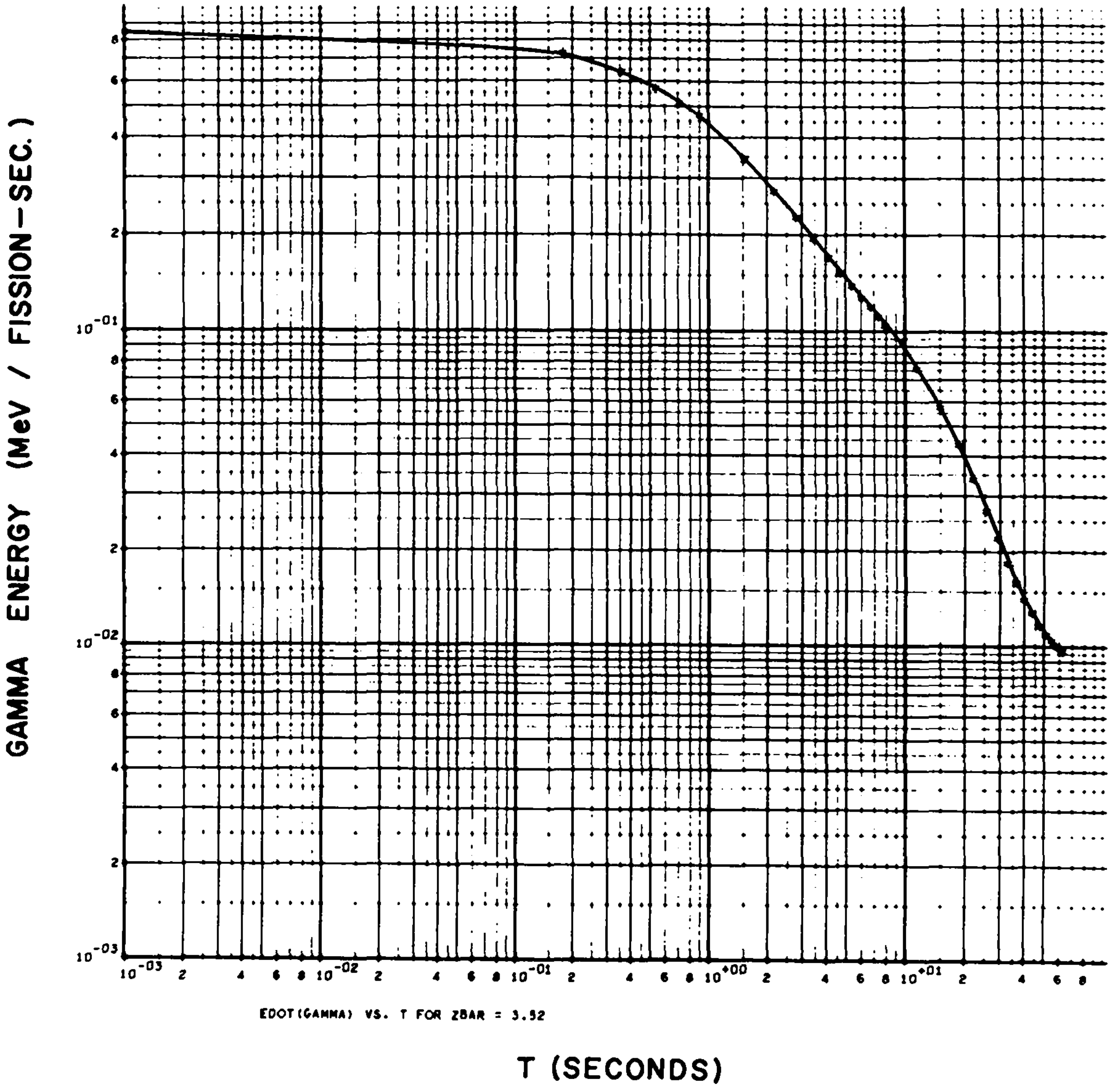GAMMA ENERGY (MeV / FISSION-SEC.)
8
6
4
2
10^-01
8
6
4
2
10^-02
8
6
4
2
10^-03
10^-03
2
4
6
8
10^-02
2
4
6
8
10^-01
2
4
6
8
10^+00
2
4
6
8
10^+01
2
4
6
8
EDOT(GAMMA) VS. T FOR ZBAR = 3.52
T (SECONDS)

Fig. 3. U^{235} + n.

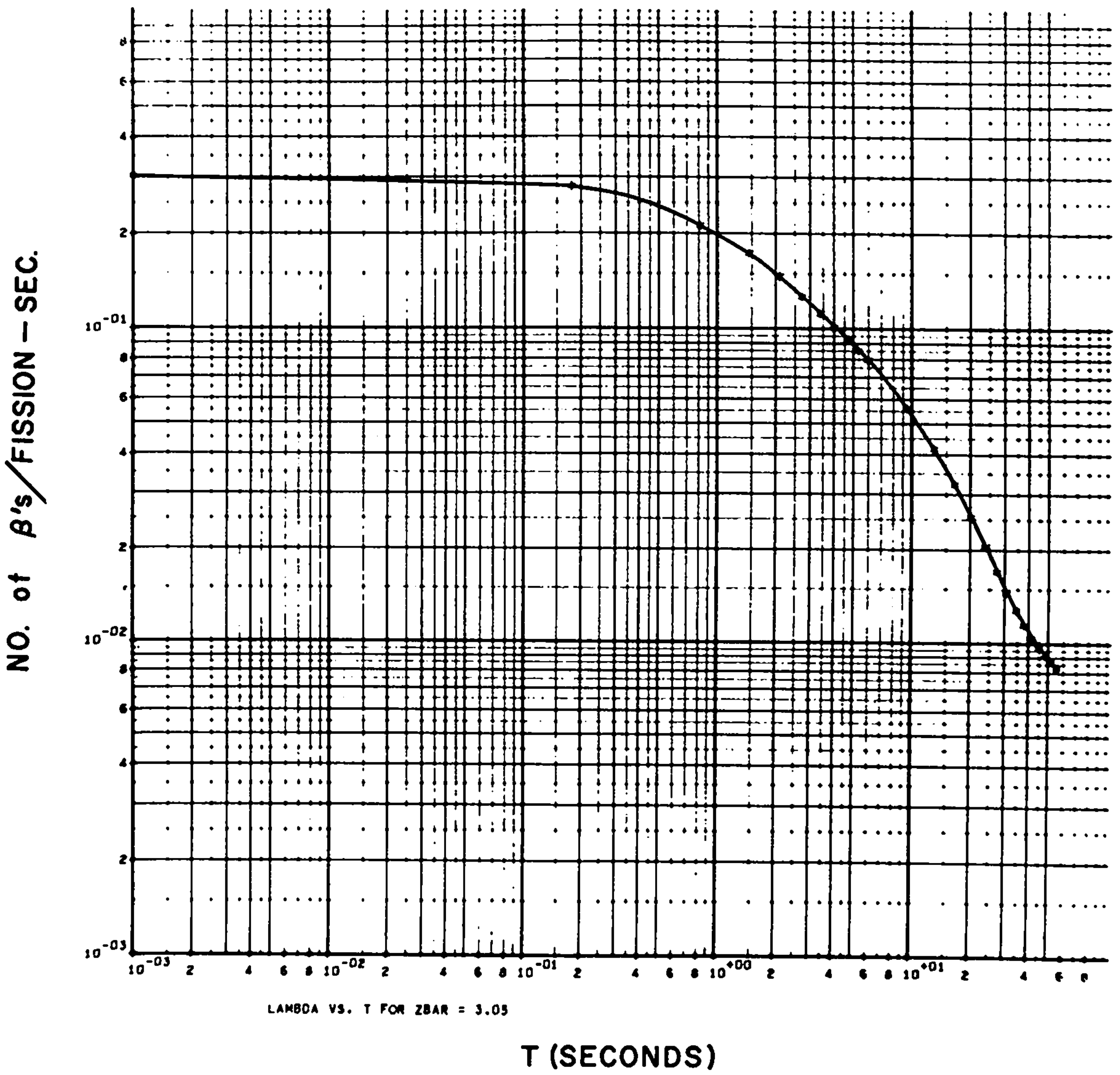
NO. of β's/FISSION — SEC.
8
6
4
2
10-01
8
6
4
2
10-02
8
6
4
2
10-03
10-03
2
4
6
8
10-02
2
4
6
8
10-01
2
4
6
8
10+00
2
4
6
8
10+01
2
4
6
8
LAMBDA VS. T FOR ZBAR = 3.05
T (SECONDS)

Fig. 4. $U^{233} + n$.

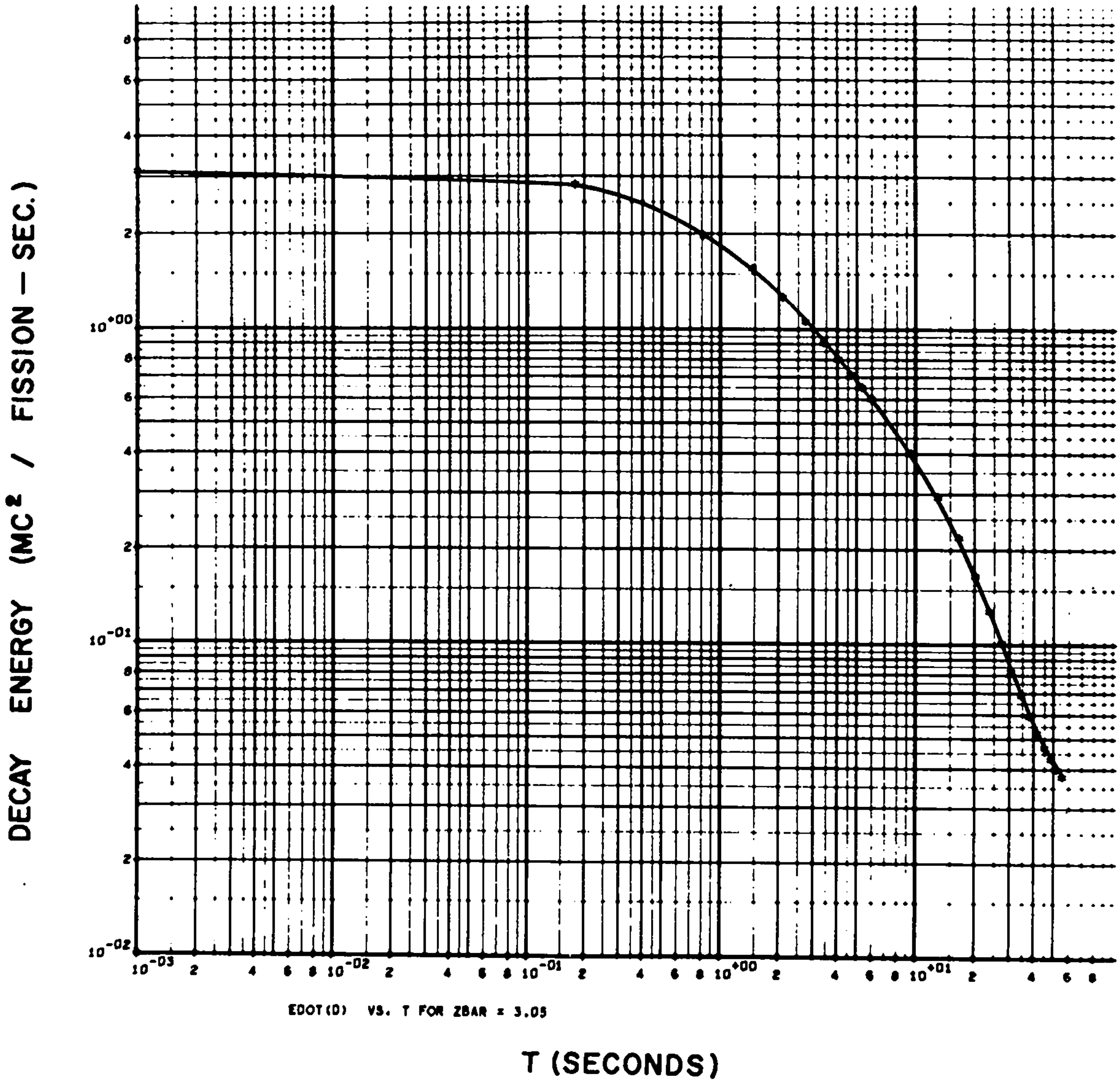
DECAY ENERGY (MC² / FISSION — SEC.)
8
6
4
2
10⁺⁰⁰
8
6
4
2
10⁻⁰¹
8
6
4
2
10⁻⁰²
10⁻⁰³ 2 4 6 8 10⁻⁰² 2 4 6 8 10⁻⁰¹ 2 4 6 8 10⁺⁰⁰ 2 4 6 8 10⁺⁰¹ 2 4 6 8
EDOT(D) VS. T FOR ZBAR = 3.05
T (SECONDS)

Fig. 5. U^{233} + n.

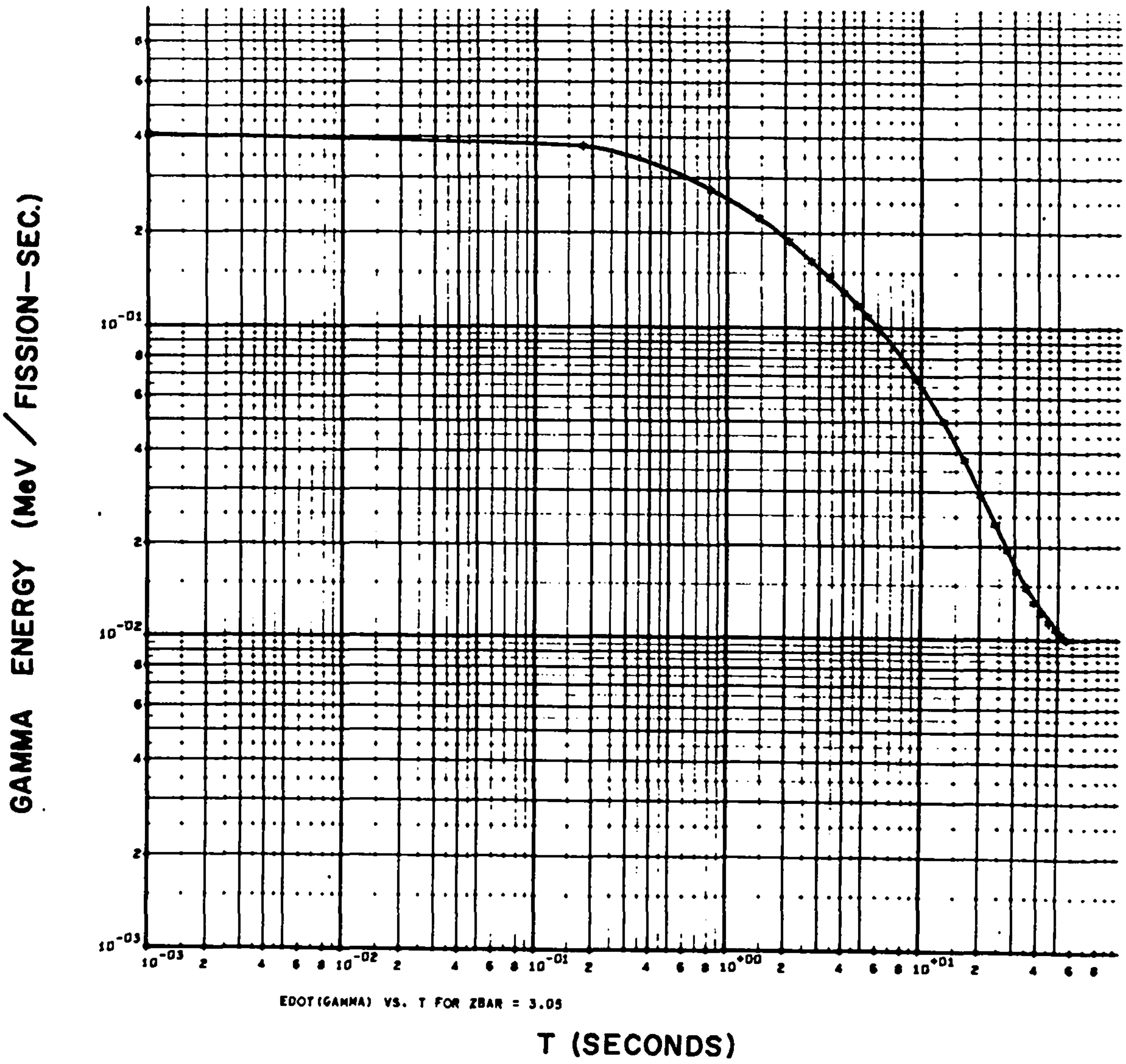
GAMMA ENERGY (MeV / FISSION-SEC.)
8
6
4
2
10-01
8
6
4
2
10-02
8
6
4
2
10-03
10-03
2
4
6
8
10-02
2
4
6
8
10-01
2
4
6
8
10+00
2
4
6
8
10+01
2
4
6
8
EDOT(GAMMA) VS. T FOR ZBAR = 3.05
T (SECONDS)

Fig. 6. U^{233} + n.

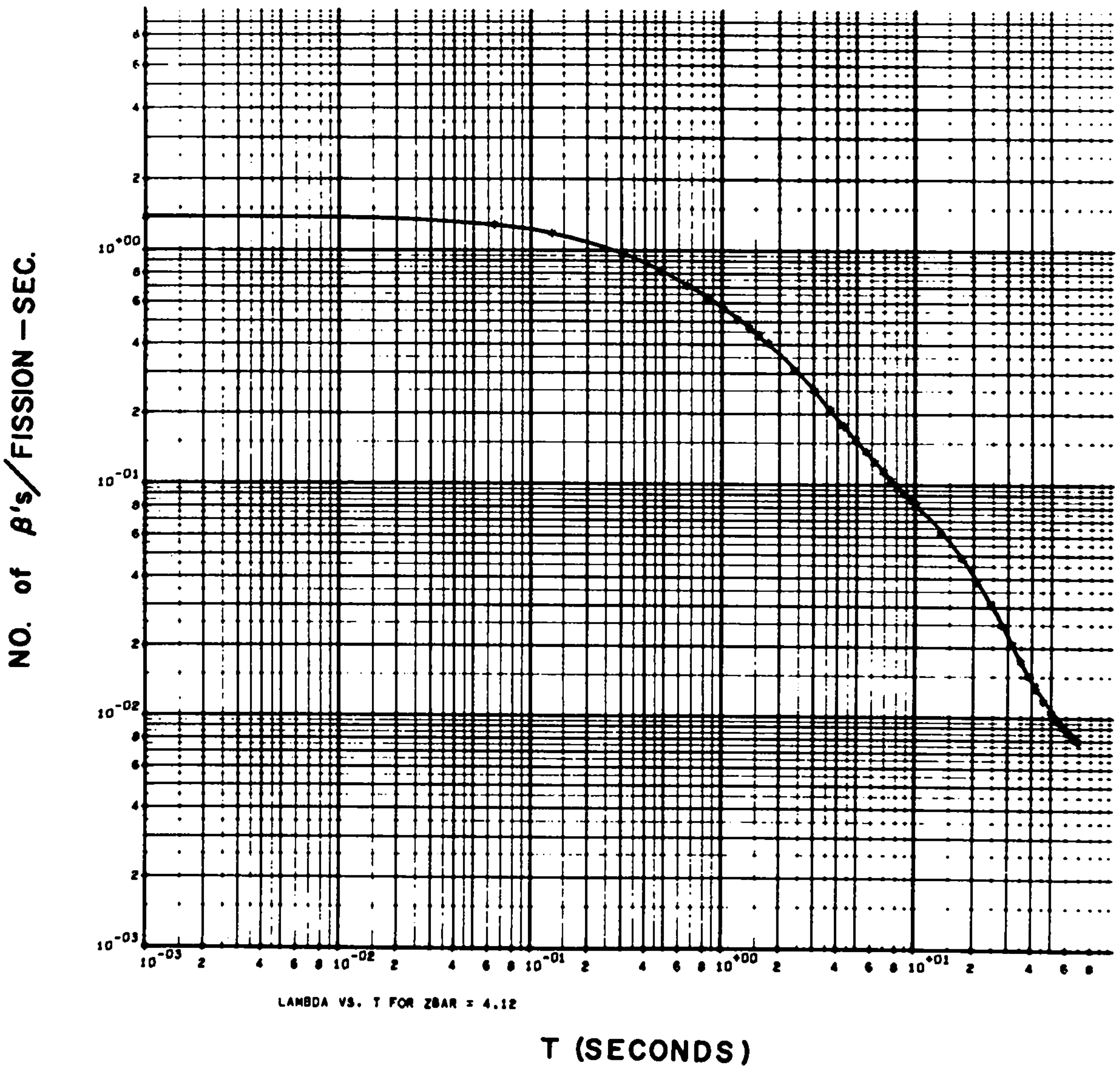
NO. of β's/FISSION-SEC.
8
6
4
2
10+00
8
6
4
2
10-01
8
6
4
2
10-02
8
6
4
2
10-03
10-03
2
4
6
8
10-02
2
4
6
8
10-01
2
4
6
8
10+00
2
4
6
8
10+01
2
4
6
8
LAMBDA VS. T FOR ZBAR = 4.12
T (SECONDS)

Fig. 7. U^{238} + n.

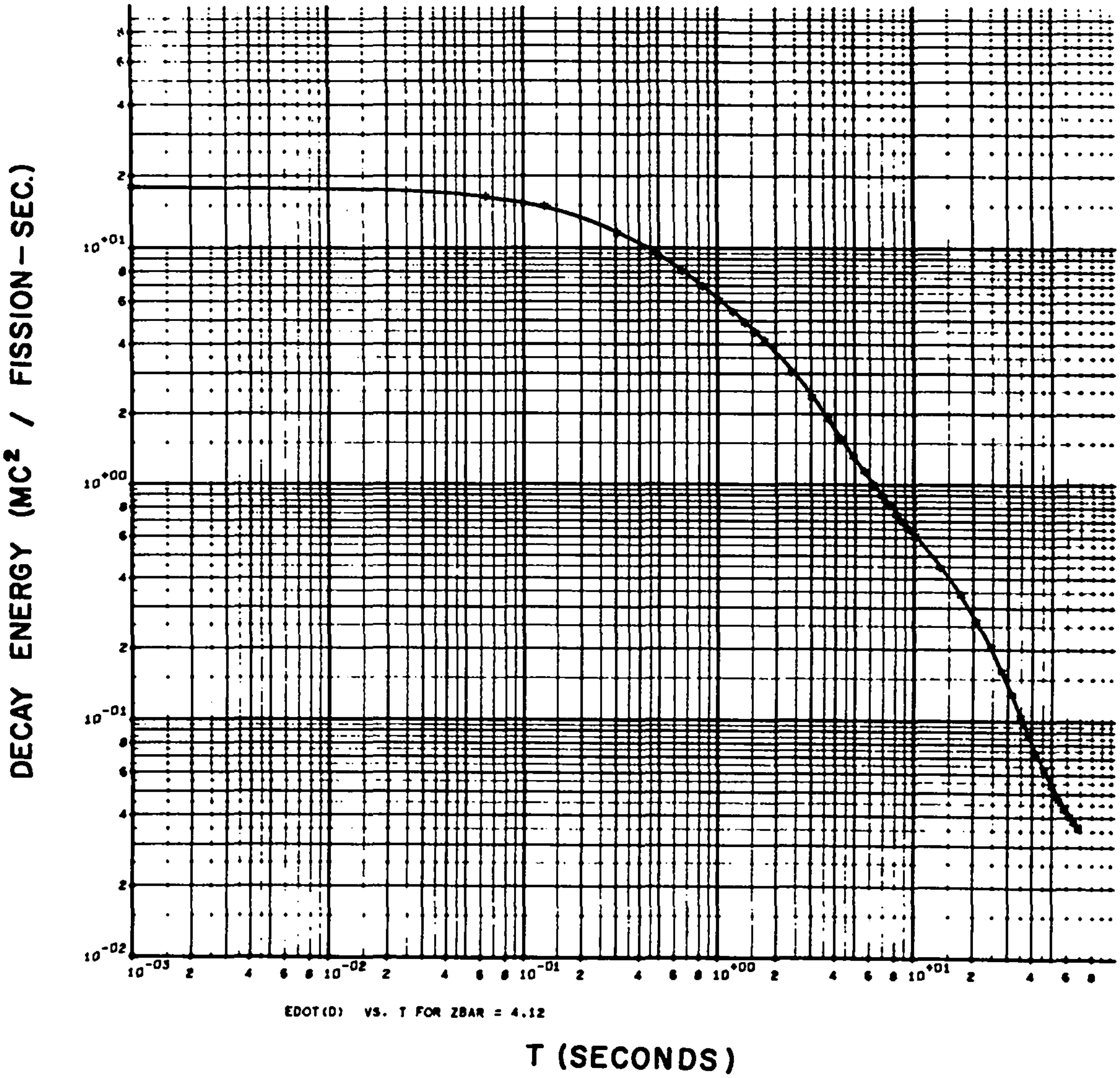DECAY ENERGY (MC² / FISSION-SEC.)
10+01
10+00
10-01
10-02
8
6
4
2
10-03
10-02
10-01
10+00
10+01
EDOT(D) VS. T FOR ZBAR = 4.12
T (SECONDS)

Fig. 8. U^{238} + n.

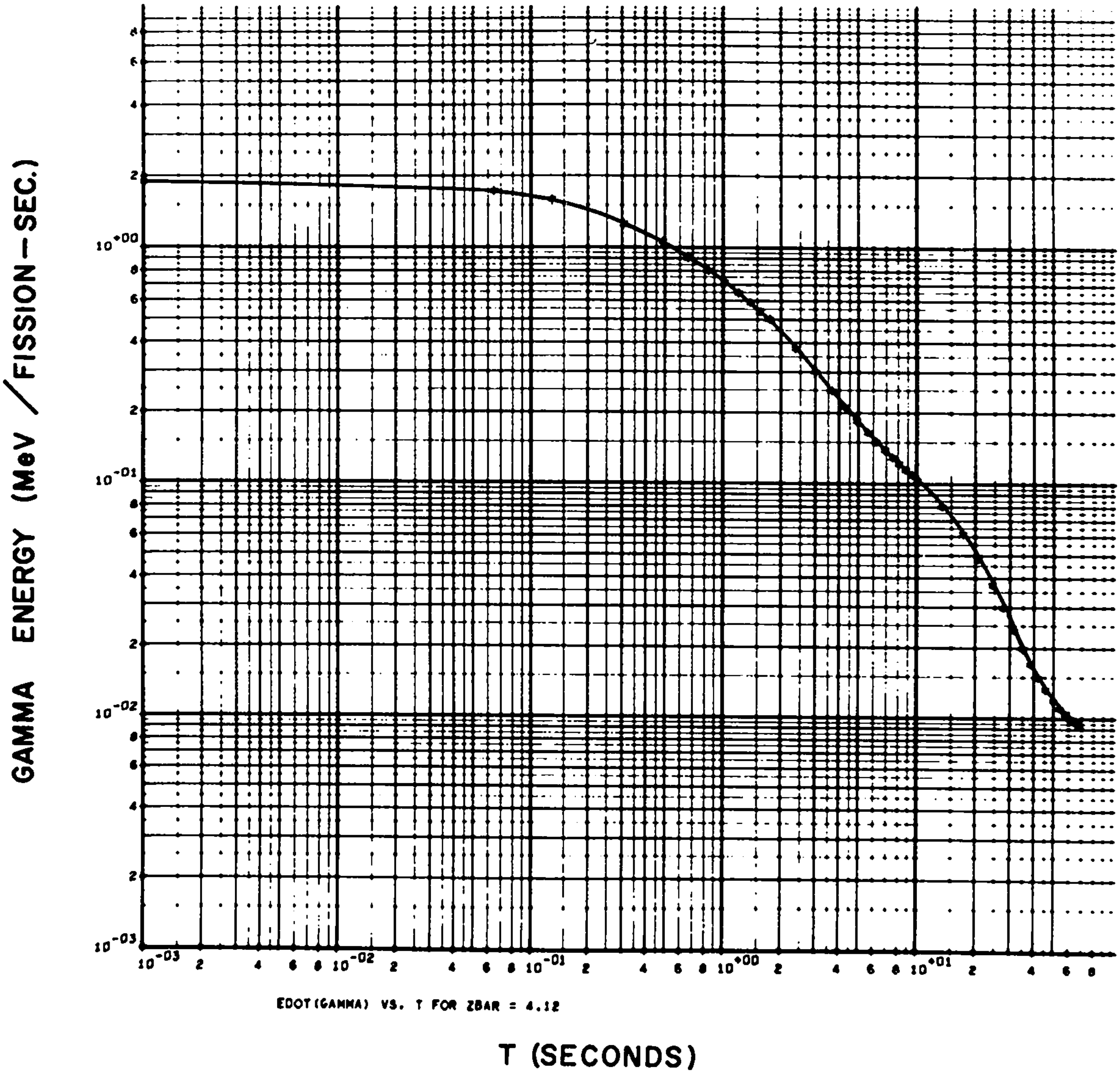
GAMMA ENERGY (MeV /FISSION-SEC.)
8
6
4
2
10^+00
8
6
4
2
10^-01
8
6
4
2
10^-02
8
6
4
2
10^-03
10^-03
2
4
6
8
10^-02
2
4
6
8
10^-01
2
4
6
8
10^+00
2
4
6
8
10^+01
2
4
6
8
EDOT(GAMMA) VS. T FOR ZBAR = 4.12
T (SECONDS)

Fig. 9. U^{238} + n.

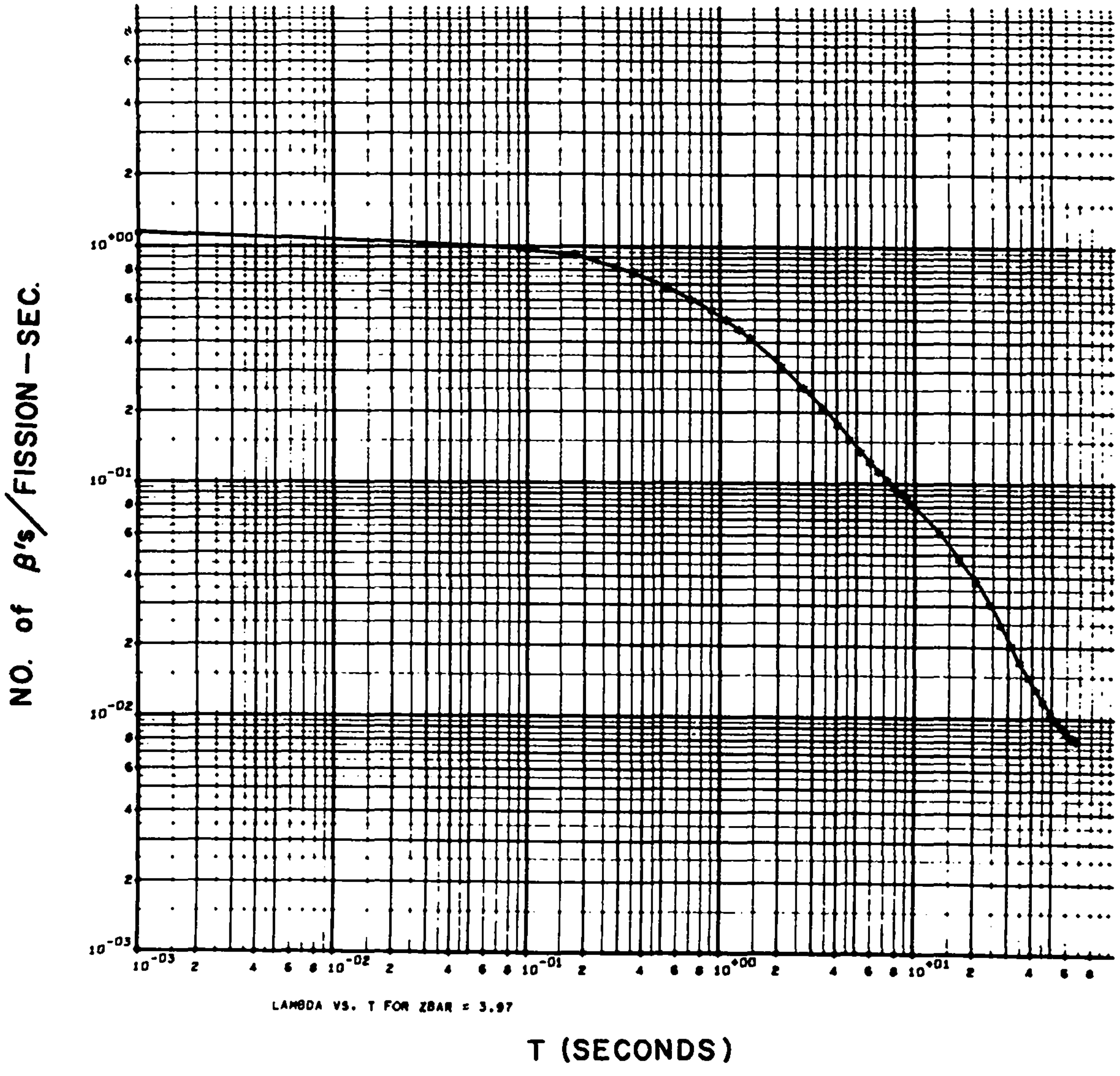

Fig. 10. Th^{232} + n.

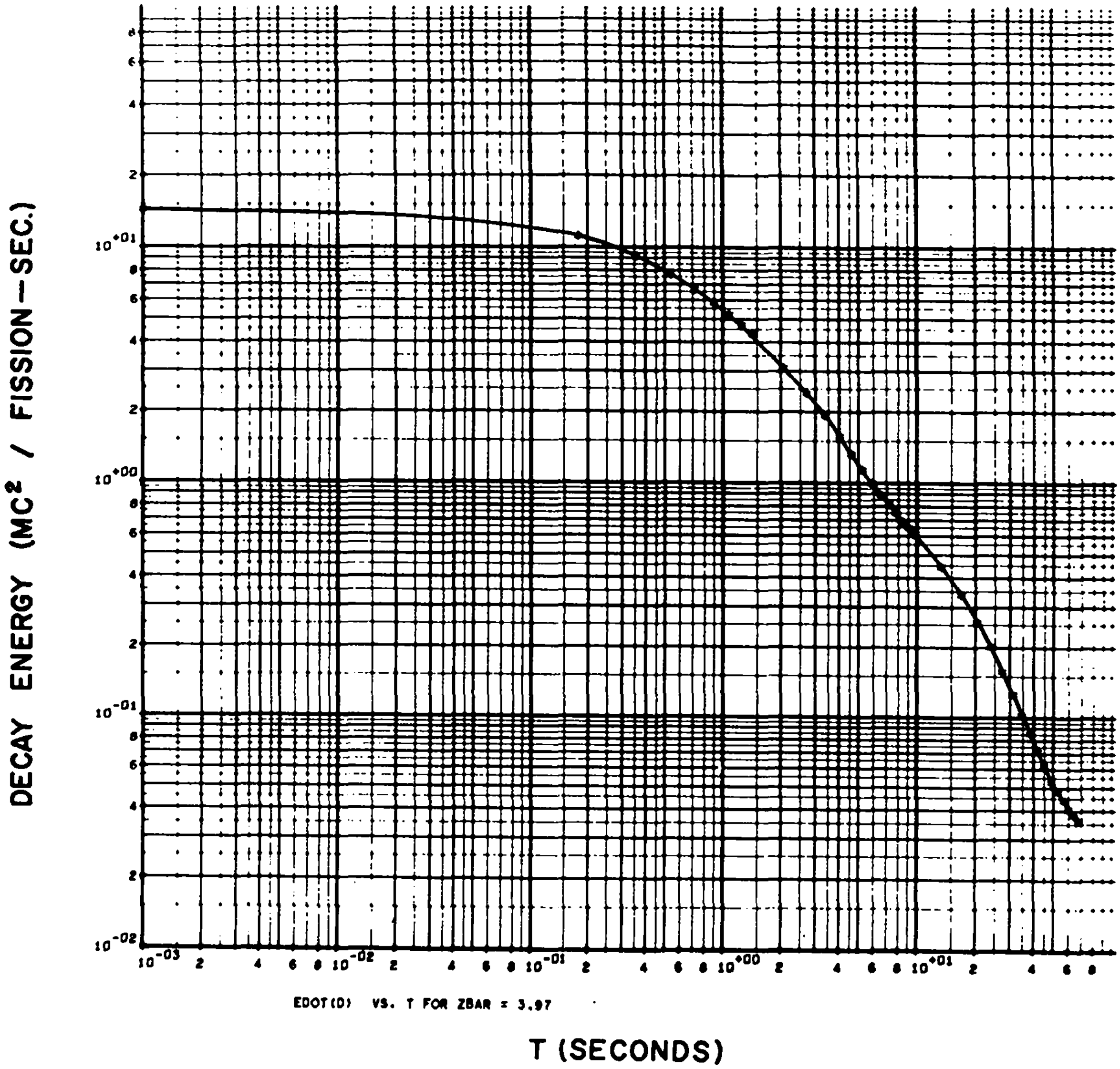

Fig. 11. $Th^{232} + n$.

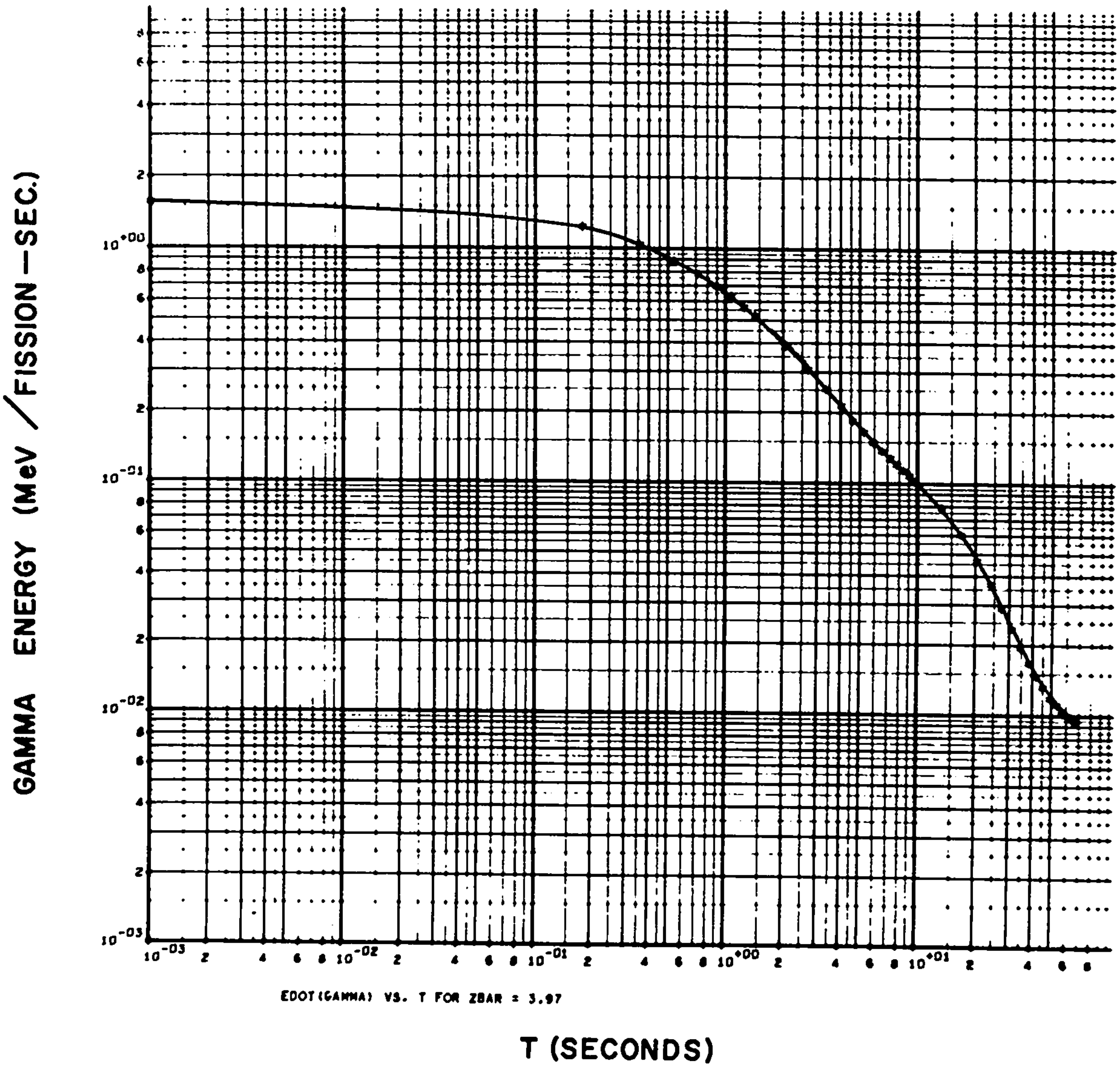

Fig. 12. Th^{232} + n.

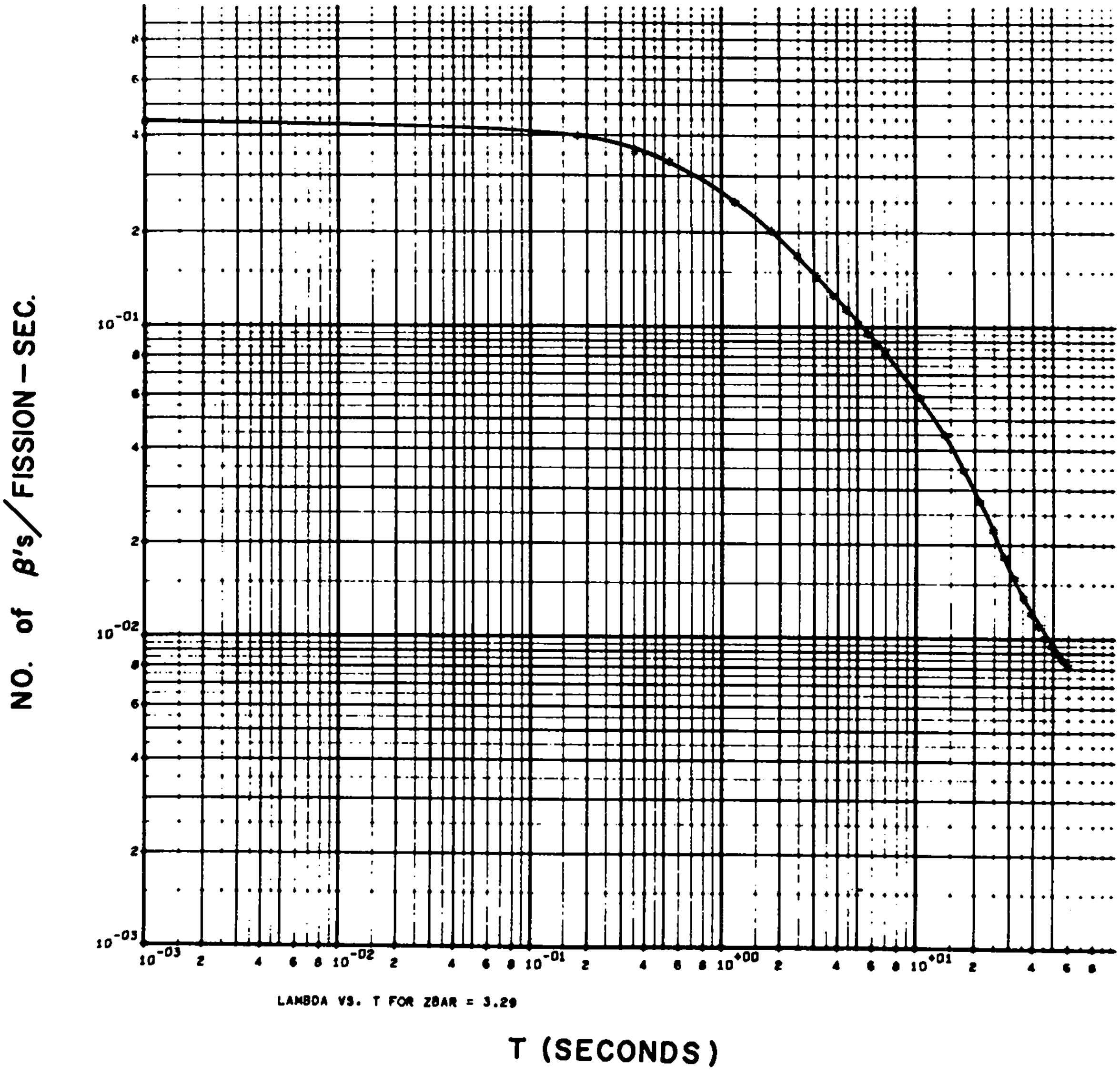

Fig. 13. Pu^{239} + n.

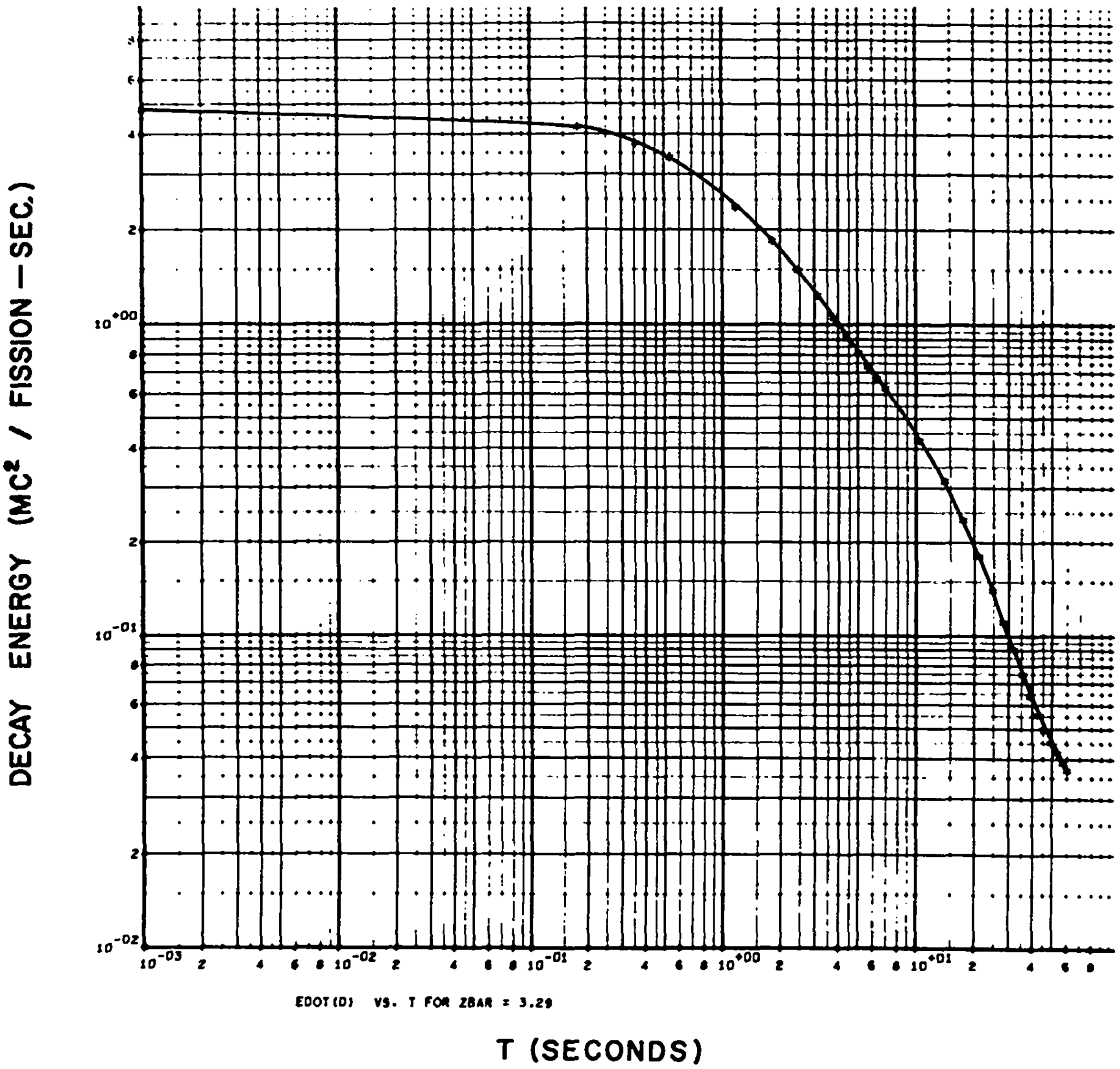

Fig. 14. Pu^{239} + n.

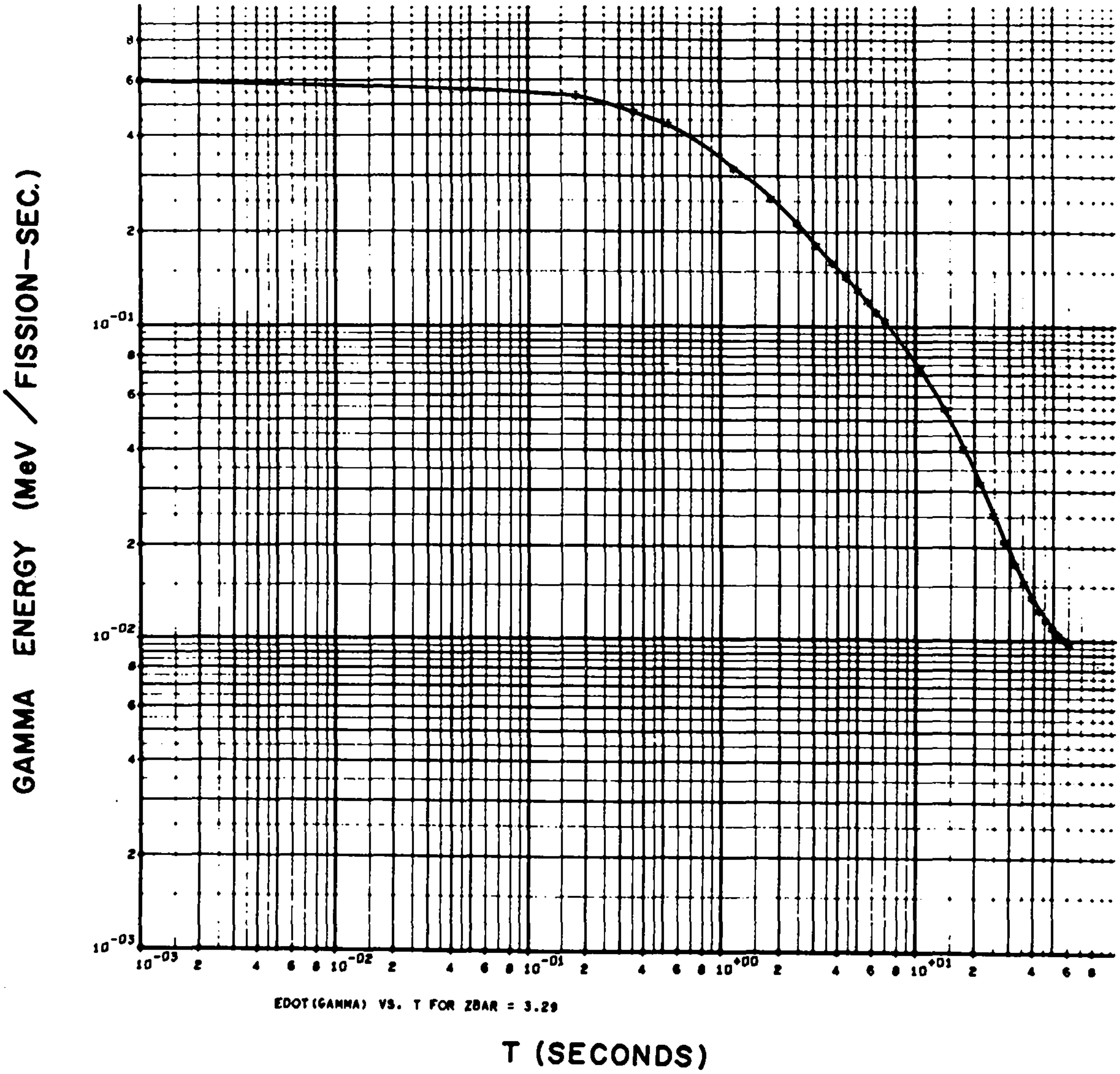

Fig. 15. Pu^{239} + n.

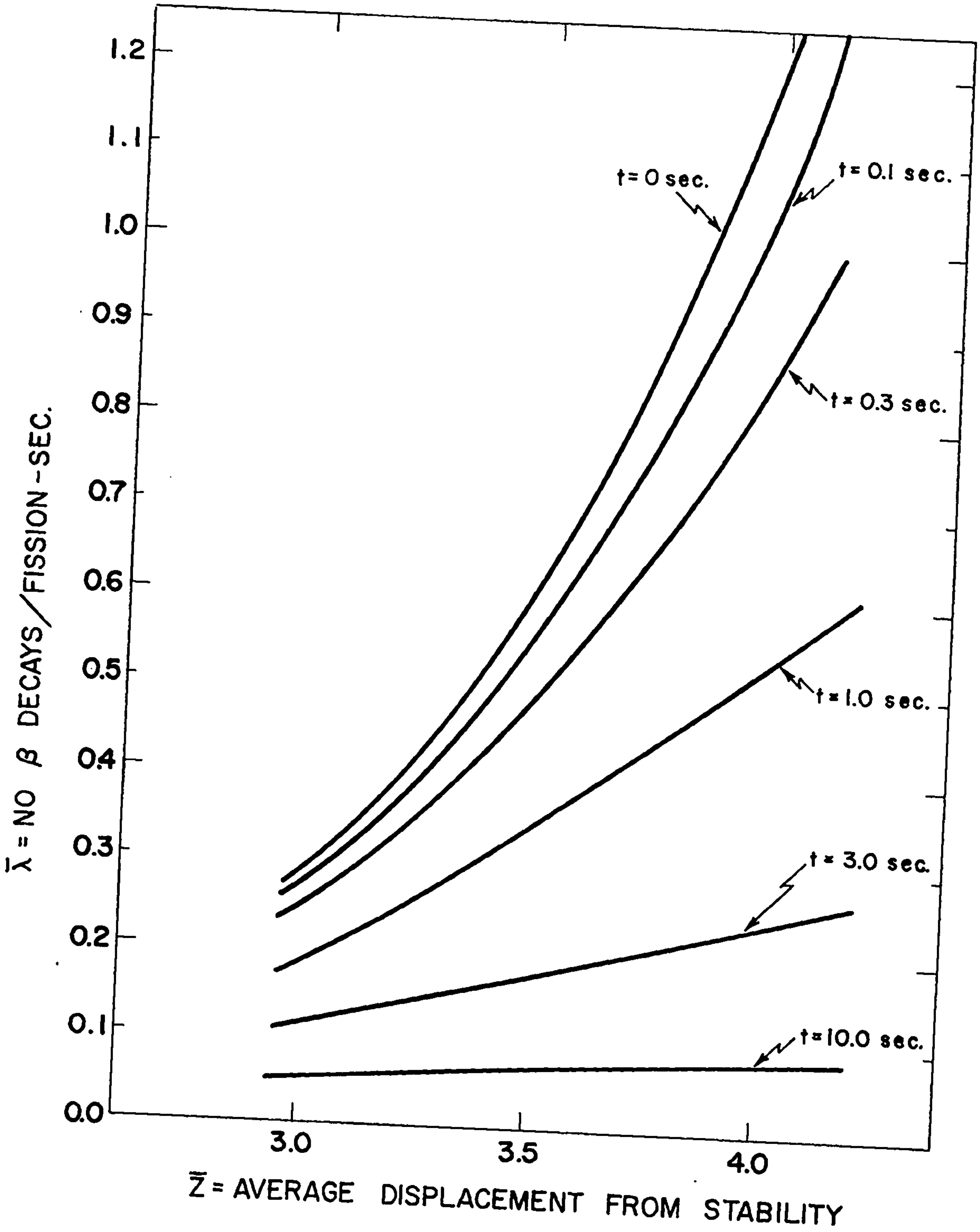

Fig. 16. $\bar{\lambda}$ vs $\bar{z}$ at various times.

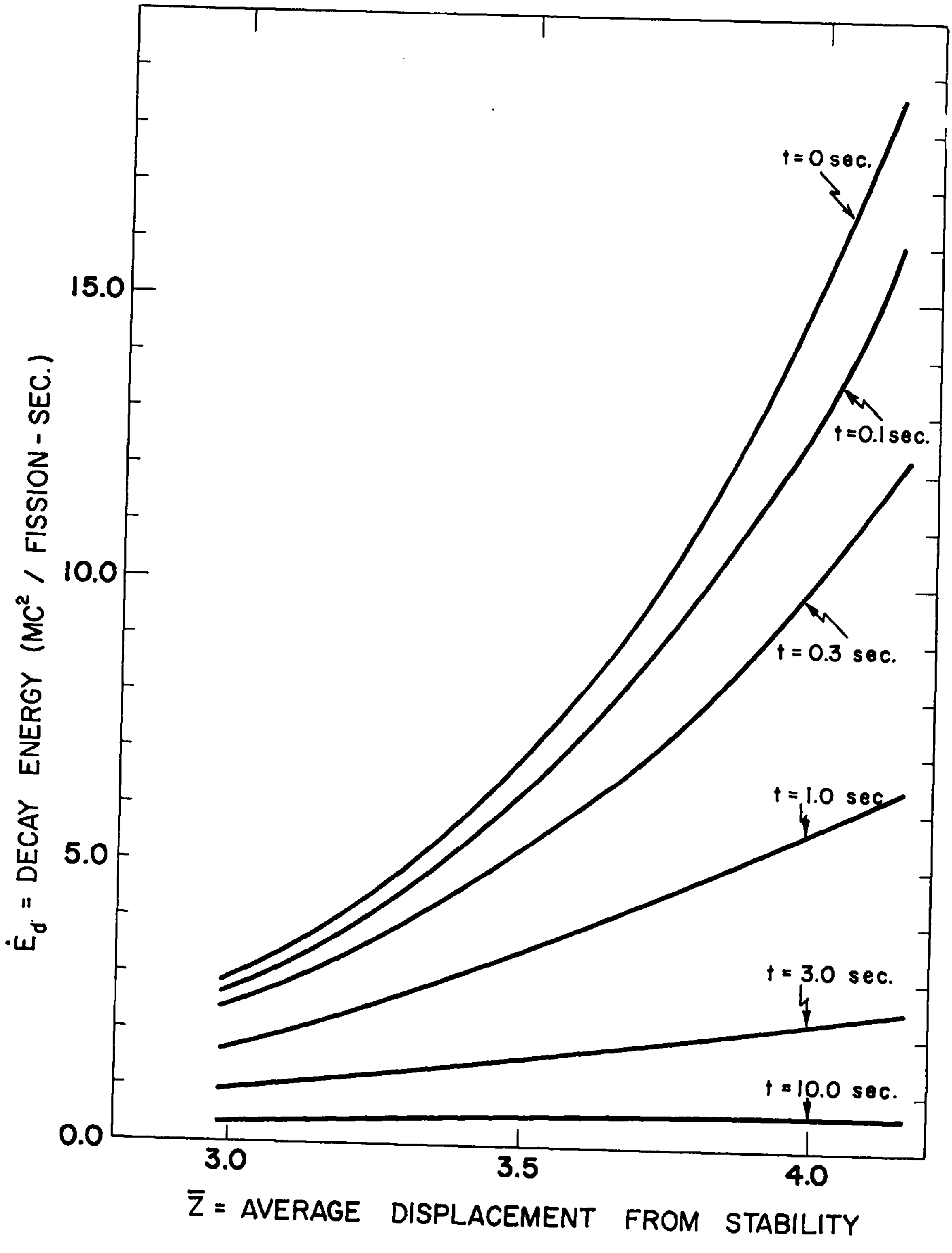

Fig. 17. $\dot{E}_d$ vs $\bar{z}$ at various times.

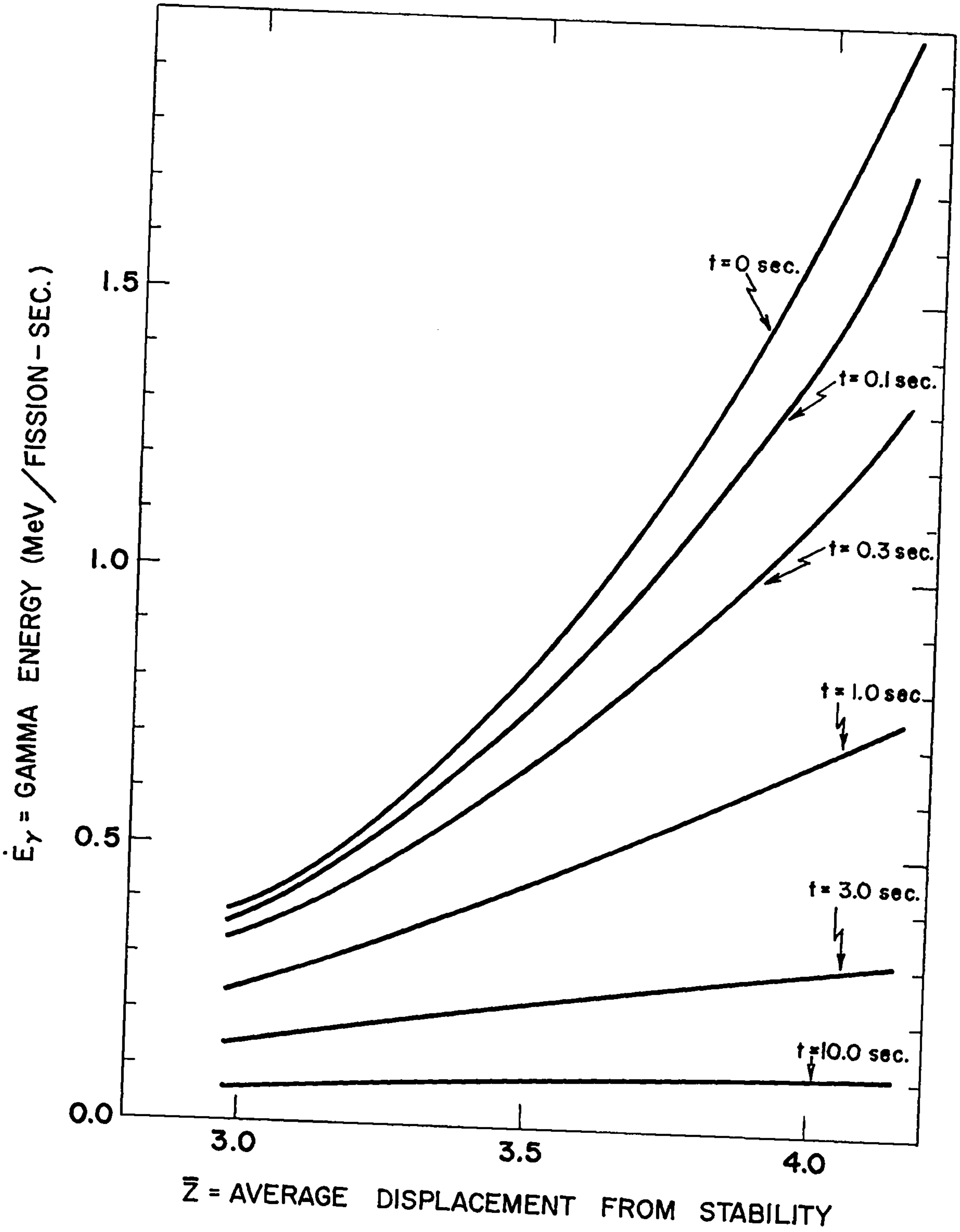

Fig. 18. $\dot{E}_\gamma$ vs $\bar{Z}$ at various times.

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7 R. B. Leachman, Proc. U. N. Intern. Conf. Peaceful Uses At. Energy, 2nd, Geneva, 15, 229 and 331 (1958).

8 Experimental evidence on the photon induced fission of U^{235}, U^{238}, Th^{232} conforms generally to the present calculations when this prescription is used, according to R. B. Walton, to be published.